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ITAE criterion based controller for buck converter

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ABSTRACT

DC-DC converters are an important class of power converters. Output voltage is subject of change due to various factors such as converter input voltage change, load change and losses in circuit elements. In order to obtain a constant output voltage, a controller must be added to the converter. In this paper, problem of designing controller for Buck converter in presence of circuit none idealities such as: switch resistance, inductor and capacitor series resistance, diode voltage drop and load current variations is solved. To do this, using State Space Averaging (SSA), small signal model of converter is obtained. In the next step, a controller is designed based on Integral of Time multiplied by Absolute Error (ITAE) performance index. Controller performance is tested with the aid of simulation for different scenarios.

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1. Introduction

A DC-DC converter is an electronic circuit that converts a source of direct current (DC) from one voltage level to another voltage level efficiently; power level range from a few Watts up to kilo Watts.

Although using transformer for level conversion is possible in presence of AC voltages, it cannot be used for DC voltage level conversion due to fixed flux that DC voltages produce (Chapman, 2012).

Based on input-output relationship of DC-DC converters 3 type of converter is possible:

- 1. Buck converter: V_{output} < V_{input}
- 2. Boost converter: V_{output} > V_{input}
- 3. Buck-Boost converter: Either V_{output} < V_{input} or Voutput>Vinput

Practical converters use switching techniques. Switching conversion is more power efficient (often 75% to 98%) than linear voltage regulation.

Energy taken from input source is stored temporarily in the form of magnetic field (of inductors or transformers) or electric field (of capacitors) and is released to output at different voltage.

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Today, control plays an important role in DC-DC converter systems. Most DC-DC converter circuits regulate the output voltage, too. Fig. 1 shows anatomy of a Buck converter. Where rS, rL and rC are input source resistance, inductor series resistance and capacitor series resistance respectively.

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Fig. 1: Buck converter circuit

Switch S is closed for time length of D×T and is open for time length (1-D)×T.D is duty cycle and T is period of one switching cycle. If we neglect the rS, rL and rC, i.e. rS=rL=rC=0, then in the steady state condition input-output relation is as in Eq. 1 (Hart, 2010; Mohan et al., 2002):

$$Vo = D.Vs \tag{1}$$

Buck converter can works in 2 modes:

Continuous Conduction Mode (CCM): Inductor current (I_L) is greater than zero during each switching cycle.

Discontinuous Conduction Mode (DCM): Inductor current goes to zero during part of the switching cycle.

2. Buck converter dynamic model

In order to design a controller for the Buck circuit mentioned before, a dynamic model must be extracted.

There are two methods which can be used to find the small signal dynamic model of a converter. These methods are:

- 1. Circuit averaging,
- 2. State space averaging (SSA).

State Space Averaging is used here. Switch SW in Fig. 1, can be either on or off. Figs. 2 and 3 shows the circuit when SW is on and off respectively.



Fig. 2: Circuit for SW=on .Diode is reverse biased

 $\begin{cases} \frac{d}{dt} \begin{pmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} \end{pmatrix} = \begin{vmatrix} \frac{r_s + r_{SW} + r_L + r_C}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{vmatrix} \times \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} r_C & \frac{1}{L} & 0 \\ -\frac{1}{C} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} i_0 \\ v_S \\ v_\gamma \end{bmatrix} \\ v_O = \begin{bmatrix} r_C & 1 \end{bmatrix} \times \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} -r_C & 0 & 0 \end{bmatrix} \times \begin{bmatrix} i_0 \\ v_S \\ v_\gamma \end{bmatrix} \end{cases}$



Fig. 3: Circuit for SW=off .Diode is forward biased

As shown in Figs. 2 and 3, load resistor is replaced with a current source I_0 . This replacement let us to study effect of load variation on the close loop response. V_{γ} and rD shows diode voltage drop and diode series resistance respectively. When switch is closed, state space equation is Eq. 2. When switch is opened, state space equation is Eq. 3. Eq. 2, is valid for an interval of length D×T and Eq. 3, is valid for an interval of length (1-D)×T. Averaging can be used in order to combine the two equations into one.

Process of averaging is nothing more than multiplying Eq. 2 and 3 by d and 1-d respectively and adding corresponding sides of equations together. It is important to note that $d = D + \overline{d}$, where D is steady state duty ratio and \overline{d} is small signal variation of duty ratio.

(3)

$$\begin{cases} \frac{d}{dt} \begin{pmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} \end{pmatrix} = \begin{bmatrix} \frac{r_L + r_D + r_C}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \times \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} r_C & 0 & -\frac{1}{L} \\ -\frac{1}{C} & 0 & 0 \end{bmatrix} \times \begin{bmatrix} i_0 \\ v_S \\ v_\gamma \end{bmatrix} \\ v_0 = \begin{bmatrix} r_C & 1 \end{bmatrix} \times \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} -r_C & 0 & 0 \end{bmatrix} \times \begin{bmatrix} i_0 \\ v_S \\ v_\gamma \end{bmatrix} \end{cases}$$

It is a standard notation in control theory to show states variables with "x", control input with "u" and output variables with "y". Based on this notation we obtain Eq. 4. State, control input and output is replaced with their corresponding large signal and small signal components (Eq. 5). In steady state, $\frac{d}{dt}(X) = 0$. This can be used to find steady state values of I_L and V_C. So, small signal model can be written as Eq. 6.

$$d \times \begin{cases} \frac{d}{dt}(x) = A_1 \cdot x + B_1 \cdot u \\ y = C_1 \cdot x + D_1 \cdot u \end{cases} \quad (1 - d) \times \begin{cases} \frac{d}{dt}(x) = A_2 \cdot x + B_2 \cdot u \\ y = C_1 \cdot x + D_1 \cdot u \end{cases}$$
(4)

$$\begin{cases} \frac{d}{dt}(X+\bar{x}) = A_1D + A_1\bar{d} + A_2(1-D) - A_2\bar{d})(X+\bar{x}) + B_1D + B_1\bar{d} + B_2(1-D) - B_2\bar{d})(U+\bar{u}) \\ Y+y = C_1D + C_1\bar{d} + C_2(1-D) - C_2\bar{d})(X+\bar{x}) + D_1D + D_1\bar{d} + D_2(1-D) - D_2\bar{d})(U+\bar{u}) \end{cases}$$
(5)

$$\begin{cases} \frac{d}{dt}(\bar{x}) = A_1 D + A_2 (1-D)\bar{x} + (B_1 D + B_2 (1-D)(A_1 - A_2)X + (B_1 - B_2)U \times \begin{bmatrix} \bar{u} \\ \bar{d} \end{bmatrix} \\ \bar{y} = C_1 D + C_2 (1-D)\bar{x} + (D_1 D + D_2 (1-D)(C_1 - C_2)X + (D_1 - D_2)U \times \begin{bmatrix} \bar{u} \\ \bar{d} \end{bmatrix} \end{cases}$$
(6)

In this paper, a Buck converter with the following parameters is considered:

Vs = 50 v $rS = 1 \Omega$ $rSW = 100 m\Omega$ D = 0.4 $V\gamma = 0.8 v$ $rD = 1 m\Omega$ $L = 400 \mu H$ $rL = 20 m\Omega$ $C = 100 \mu H$ $rC = 50 m\Omega$ $f_{switching} = 20 KHz$

Extraction of transfer functions from equation 6 by hand is cumbersome to some extent. So a Matlab[®] program is written to do the job as follows:

%%circuit parameters Vs=50; rs=1; D=.4; L=400e-6; rl=20e-3; C=100e-6; rc=50e-3: rsw=100e-3; Vg=.8;rd=1e-3; f=20e3; R=20; *lo=1;* T=1/f;%%model for 0<t<DT A1=(-(rs+rsw+rl+rc)/L -1/L;1/C 0); *B1=(rc/L 1/L 0;-1/C 0 0);* C1=(rc 1); 1=(-rc 0 0); %model for DT<t<T A2=(-(rl+rd+rc)/L -1/L;1/C 0); 25 B2=(rc/L 0 -1/L;-1/C 0 0); C2=(rc 1); D2=(-rc 0 0); %%averaging U=(Io;Vs;Vg);*X*=-*inv*(*D***A*1+(1-*D*)**A*2)*((*D***B*1+(1-325*D*)**B*2)**U*); %X is Steady state inductor current(IL) and capacitor voltage(Vc) A=D*A1+(1-D)*A2; B=(D*B1+(1-D)*B2 (A1-A2)*X+(B1-385B2)*U); C=(D*C1+(1-D)*C2); DD=(D*D1+(1-D)*D2 (C1-C2)*X+(D1-D2)*U);%DD is used as a symbol %for State space D matrix to %avoid confusion with Duty ratio %of converter(D) 46 sys=ss(A,B,C,DD); tf(sys) bode(sys)

Running the code, gives the following results shown in Fig. 4.

$$X = \begin{bmatrix} 1\\ 19.0594 \end{bmatrix} \quad U = \begin{bmatrix} 1\\ 50\\ 0.8 \end{bmatrix}$$

$$\frac{\tilde{v}_o(s)}{2} - \frac{-.05 \times s^2 - 1.006 \times 10^4 \times s - 1.151 \times 10^7}{2}$$

$\tilde{\iota}_o(s)$	-	$s^2 + 1277 \times s + 2.5 \times 10^7$
	$\tilde{v}_o(s)$	$50s + 10^7$
	$\overline{\tilde{v}_s(s)}$ =	$\overline{s^2 + 1277 \times s + 2.5 \times 10^7}$
	$\tilde{v}_o(s)$	$-75s - 1.5 \times 10^{7}$
	$\frac{1}{\tilde{v}_{\gamma}(s)}$	$\overline{s^2 + 1277 \times s + 2.5 \times 10^7}$
	$\tilde{v}_o(s)$	$6213s + 1.243 \times 10^9$
	$\tilde{d}(s)$	$s^2 + 1277 \times s + 2.5 \times 10^7$

Fig. 4: Result of running the code

Using output voltage equation for the ideal converter (i.e. $rS=rSW=V\gamma=rD=rL=rC=0$), we expect output voltage and output ripple to be Eqs. 7 and 8 (Hart, 2010; Mohan et al., 2002):

$$Vo = D \times Vs = 0.4 \times 50 = 20$$
(7)
$$\frac{\Delta V_0}{D} = \frac{1 - D}{D} = \frac{0.6}{0.6} = 4.69 \times 10^{-3}$$

$$V_0 = \frac{1}{8 \times L \times C \times f^2} = \frac{1}{8 \times 400 \mu \times 100 \mu \times (20k)^2} = \frac{1}{4.09} \times 100$$

$$\Delta V_0 = V_0 \times 4.69 \times 10^{-3} = 234.5 \ mV$$
(8)

Although, above formulas are not correct for the non-ideal converter (i.e. a converter with non-ideal components), it give us some idea.

Steady state inductor current and capacitor voltage can be calculated by equating large signal steady state portion of state equation to zero, i.e. $\frac{d}{dt}(X) = 0$. This is done in line 30-32 of Matlab[®] code. After running code we get X=(1 19.0594)^T.

Fig. 5 shows simulation diagram of circuit without controller in Matlab[®]. Load is a 1A current source.

Inductor current and capacitor voltage are as shown in Figs. 6 and 8 respectively.

Fig. 7 shows inductor current in steady state. As seen in Fig. 7 minimum of current is about 0.2 A, so converter is in Continues Conduction Mode (CCM). Capacitor ripple voltage is as shown in Fig. 9.

3. Controller design

Small signal model of converter is extracted in equation in Fig. 4. Here we need a controller which changes duty ratio in order to set the output voltage. Variable under control is load voltage and control input is duty ratio.

In order to obtain pulses for switch, controller output signal must be converted into a train of on off pulses. This job can be done with the aid of Pulse Width Modulation (PWM) technique. PWM techniques are beyond the scope of this paper. An interested reader is can refer to Hart (2010), Mohan et al. (2002), and Ericson and Maksimovic (1997).

Where Vp is triangular reference amplitude. If Vp= 1 then its transfer function reduces to $\frac{1}{1} = 1$ and has no effect on the overall open loop transfer function. Here it is assumed Vp= 1. Converter small signal transfer function is Eq. 9.

$$\frac{\bar{\nu}_0}{\bar{d}} = \frac{6213s + 1.243 \times 10^9}{s^2 + 1277s + 2.5 \times 10^7} \tag{9}$$

PWM block transfer function is $\frac{1}{v_n}$ (Hart, 2010),

Pole-Zero diagram of Eq. 9 is drawn in Fig. 10.



Fig. 5: Simulation diagram of system without controller



Fig. 6: Inductor instantaneous current (blue) and average current (red).Average current goes toward 1A



Fig. 7: Inductor current in steady state. IL(max)=1.793 A and IL(min)=0.185 A



Fig. 8: Capacitor voltage. Average steady state value is 18.85 v





Transfer function has a zero at $^{-2\times10^5}$ and poles at $^{-638.5\pm4959.1j}$.Cross over frequency and phase margin (Dorf and Bishop, 2001) of this transfer

function is $3.59 \times 10^4 \frac{rad}{s} = 5.714 KHz$ and 12.2° , respectively. Frequency response is shown below in Fig. 11.



Fig. 11: Frequency response of open loop transfer function of Eq. 9

A common controller for Buck converter is type 2 error amplifier, shown in Fig. 12.



Fig. 12: Type 2 error amplilifier and its frequency response.

Type 2 error amplifier has the following small signal transfer function (Eq. 10):

$$\frac{\nu_c}{\nu_o} = -\frac{s + \frac{1}{R_2 C_2}}{R_1 C_2 s \left(s + \frac{C_1 + C_2}{R_2 C_1 C_2}\right)} = -\frac{s + \omega_p}{R_1 C_2 s (s + \omega_p)}$$
(10)

In order to find values of R_1 , R_2 , C_1 , C_2 a design methology based on phase margin can be used.

An interested reader is referred to Dorf and Bishop (2001) and Hart (2010) in order to learn the method.

Here, a method based on optimal control theory is used to find controller parameters.

Fig. 13 shows overall configuration of close loop system.

First of all, close loop transfer function must be extracted. General form of a feedback control system is shown in Fig. 14.



Fig. 13: Close loop system configuration



Fig. 14: General feedback control system and its close loop transfer function

In Fig. 14, G(s) and H(s) shows controller transfer function(Type 2 error amplifier) and plant transfer function (Buck converter dynamics) respectively. So,

$$G(s) = \frac{s + \omega_z}{R_1 C_2 s(s + \omega_p)} = \frac{s + \omega_z}{\alpha C_2 s(s + \omega_p)}$$

and

$$H(s) = \frac{\bar{v}_0}{\bar{d}} = \frac{6213s + 1.243 \times 10^9}{s^2 + 1277s + 2.5 \times 10^7}$$

Negative sign of Eq. 10 is put in the summing point so G(s) is positive part of Eq. 10. Assume that

$$B(s) = 6213s + 1.243 \times 10^9$$

and

$$A(s) = s^2 + 1277s + 2.5 \times 10^7$$

So close loop transfer function is Eq. 11.

$$H_{close-loop} = \frac{B(s) \times \alpha \times (s + \omega_z)}{A(s) \times (s + p) + B(s) \times \alpha \times (s + z)}$$
(11)

Close loop characteristic equation, i.e. denominator of Eq. 11, is Eq. 12.

 $s^{4} + (1000 + \omega_{p})s^{3} + (1000\omega_{p} + 6250\alpha)s^{2} + (2.5 \times 10^{7} + 6250 \times \alpha \times \omega_{z} + 1.25 \times 10^{9} \times \alpha)s + (2.5 \times 10^{7}\omega_{p} + 1.25 \times 10^{9} \times \alpha \times \omega_{z}$ (12)

Roots of Eq. 12 gives close loop poles. Design parameters are: ω_z , ω_p and α .

A performance index is a quantitative measure of the performance of a system and is choose so that emphasis is given to the important system specifications (Dorf and Bishop, 2001). Various performance indexes can be defined. Some of them are: Integral of square error (ISE), Integral of the absolute magnitude of the error (IAE) and Integral of Time multiplied by Absolute Error (ITAE). An interested reader is referred to Dorf and Bishop (2001) in order to gather more information about this performance indexes.

In order to minimize ITAE performance index for step input, coefficients of close loop characteristic polynomial must related to each other as given in Table 1.

 Table 1: Optimum coefficients of close loop characteristics

 based on the ITAE criterion for a step input

$s + \omega_n$
$s^2 + 1.4 \omega_n s + \omega_n^2$
$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$
$s^4 + 2.1\omega_n s^3 + 3.4 \omega_n^2 s^2 + 2.7 \omega_n^3 s + \omega_n^4$
s^{5} + 2.8 $\omega_{n} s^{4}$ + 5.0 $\omega_{n} s^{3}$ + 5.5 $\omega_{n}^{3} s^{2}$ + 3.4 $\omega_{n}^{4} s$ + ω_{n}^{5}

Comparison of Eq. 12 and fourth row of Table 1 leads to Eq. 13:

$1000 + \omega_p = 2.1\omega_n$	(13-1)
$1000\omega_p + 6250\alpha = 3.4\omega_n^2$	(13-2)
$(2.5 \times 10^7 + 6250 \times \alpha \times \omega_z + 1.25 \times 10^9 \times \alpha) =$	
$2.7\omega_n^3$	(13-3)
$(2.5 \times 10^7 \omega p + 1.25 \times 10^9 \times \alpha \times \omega_z) = \omega_n^4$	(13-4)

In the above system of equation, ω_n is a known number set by the designer. There is 4 equations and 3 unknowns. We assume $\omega_n = 2\pi \times 1000$.

In order to obtain unique values for ω_z , ω_p and α , 3 equations are chosen. There are 4 different ways to choose 3 equations out of 4.0btained system of 3 equations and 3 unknowns are solved for each case separately. Following Matlab[®] code is used to solve equations; Results are given in Table 2:

syms wp wz alpha disp('solving 13-1,13-2 and 13-3') s1=solve('1000+p=2.1*(2*pi*1000)',... '1000*p+6250*alpha=3.4*(2*pi*1000)^2',... '2.5e7+6250*alpha*z+1.25e9*alpha=2.7*(2*pi*1000)^3'); s1.p %prints pole location s1.z %prints zero location s1.alpha %prints alpha disp('solving 13-2,13-3 and 13-4') s2=solve('1000*p+6250*alpha=3.4*(2*pi*1000)^2',... '2.5e7+6250*alpha*z+1.25e9*alpha=2.7*(2*pi*1000)^3',... '2.5e7*p+1.25e9*alpha*z=(2*pi*1000)^4'); s2.p s2.z s2.alpha disp('solving 13-1,13-2 and 13-4') s3=solve('1000+p=2.1*(2*pi*1000)', '1000*p+6250*alpha=3.4*(2*pi*1000)^2', '2.5e7*p+1.25e9*alpha=(2*pi*1000)^4'); s3.p s3.z s3.alpha disp('solving 13-1,13-3 and 13-4') s4=solve('1000+p=2.1*(2*pi*1000)',... '2.5e7+6250*alpha*z+1.25e9*alpha=2.7*(2*pi*1000)^3',... '2.5e7*p+1.25e9*alpha*z=(2*pi*1000)^4'); s4.p s4.z s4.alpha

Table 2: Results of solving 4 equation set							
Chosen equations	ω_{z}	ω_p	α	Close loop stability			
(13-1)	194.512	12.195	19.525				
(13-2)	×	×	×	NO			
(13-3)	10 ³	10 ³	10 ³				
(13-1)	2.354	12.195	0.529				
(13-3)	×	×	×	NO			
(13-4)	10 ³	10 ³	10 ³				
(13-1)	No	No	No				
(13-2)	Solution	Solution	Solution	-			
(13-4)	Solution	Solution	Solution				
(13-2)	2.349	130.917	0.529				
(13-3)	×	×	×	YES			
(13-4)	10 ³	10 ³	10 ³				

With the parameters given in fourth row of Table 2, close loop poles (= Eq. 12 roots) are Eq. 14.

$p_1 = -130.93 \times 10^3 s^{-1}$	
$p_2 = -0.399 \times 10^3 s^{-1}$	(14)
$p_{3.4} = (-0.294 \pm 5.450i) \times 10^3 s^{-1}$	

So, Controller transfer function is Eq. 15.

$$G(s) = \frac{s + 2.349 \times 10^3}{0.529 \times 10^3 \times s \times (s + 130.917 \times 10^3)}$$
(15)

4. Simulation results

In order to test the performance of control loop, following scenario is applied at Simulink[®] environment of Matlab[®].

At t=25 ms load current changes from 1A to 3A. At t=50 ms Vs is change from 50V to 40V. At t=75ms load changes from 3A to 1.5A and at t=100ms Vs changes from 40V to 60V.At t=250s reference voltage change from 20v to 25v. Figs. 15-20 shows the simulation results.











5. Conclusion

Buck converters are used when output voltage must be less than input voltage.Generelly, output voltage must keep relatively constant despite of changes in the load current. Feedback control must be used to acheive this goal.



Fig. 19: Effect of refrence change



Fig. 20: Controller output signal(control signal)

In this paper, problem of designing controller for Buck converter is solved with the aid of optimum coefficients of close loop charachteristic equation of ITAE criterion.

Designed controller is tested under various scenarios in Simulink® environment of Matlab®. Simulation results shows that loop is stable and has an acceptable time response.

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